

Evaluation of the curvature of a superellipse with “radius” 1 at the point (1,0)

Given any point (x,y) on the curve with x<1:

$$d = (-y, x-1)$$

$$M = \left(\frac{x-1}{2}, \frac{y}{2}\right)$$

$$C = (1-R, 0) = M + td$$

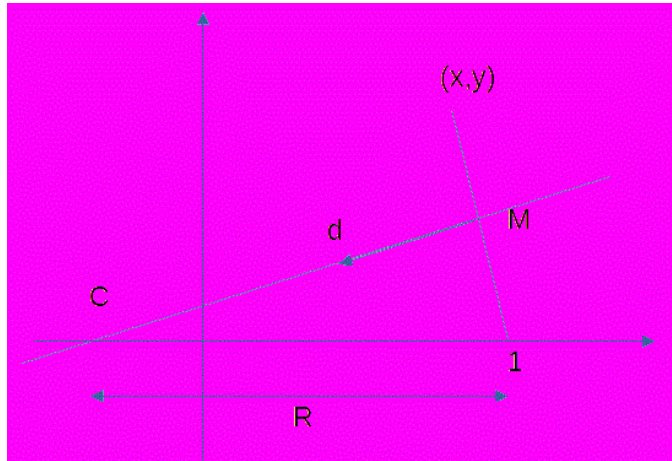
$$C = \left(\frac{x+1}{2} - ty, \frac{y}{2} + t(x-1)\right)$$

$$0 = \frac{y}{2} + t(x-1)$$

$$t = \frac{-y}{2(x-1)}$$

$$1-R = \frac{x+1}{2} + \frac{y^2}{2(x-1)}$$

$$1-R = \frac{x^2 - 1 + y^2}{2(x-1)}$$



where R is the curvature radius approximation.

Considering a superellipse with “radius” 1:

$$x^n + y^n = 1 \quad y^2 = (1-x^n)^{(2/n)}$$

$$1-R(x) = \frac{x^2 - 1 + (1-x^n)^{(2/n)}}{2(x-1)}$$

Taking the limit as x goes to 1:

$$\lim_{x \rightarrow 1} R(x) = 1 - \frac{2x - (2/n)(1-x^n)^{(2/n)-1}}{2}$$

And applying l'Hôpital rule:

$$\lim_{x \rightarrow 1} R(x) = 1 - \lim_{x \rightarrow 1} x - (1-x^n)^{(2/n)-1}$$

For n=2, as the exponent (2/n)-1 is zero for all x, then:

$$\lim_{x \rightarrow 1} R(x) = 1$$

For n>2, as the exponent (2/n)-1 is less than zero for all x, then:

$$\lim_{x \rightarrow 1} R(x) = \infty$$